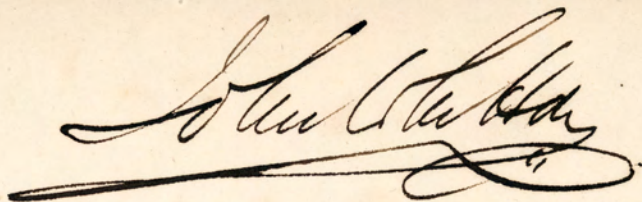


THE
STRAINS ON STRUCTURES
OF
IRON WORK.
BY
F. W. SHEILDS, M. INST. C. E.



THE STRAINS
ON
STRUCTURES OF IRONWORK;
WITH
PRACTICAL REMARKS
ON
IRON CONSTRUCTION.

BY
F. W. SHEILDS, M.INST.C.E.

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TO

CHARLES VIGNOLES, Esq., F.R.S.

CIVIL ENGINEER,

IN TOKEN OF LONG AND SINCERE FRIENDSHIP,

BY HIS FORMER PUPIL,

THE AUTHOR.

P R E F A C E.

FROM a varied experience in the practice of Civil Engineering, the subject of the present treatise appears to me one of the most difficult branches of the profession. My engagements of late years have induced me to study it with peculiar attention, as I was appointed in 1852 to assist the principal officers of the Crystal Palace Company at Sydenham, in calculating the strength and designing the details of the Company's works; and my subsequent practice, arising in a great degree from this connection, has partaken much of a similar character. I found great difficulty, however, in acquiring a complete knowledge of the strains on the several parts of iron framings, as the works of previous authors, though displaying great talent and research, have left much undetermined or obscure which is necessary to the designer of such structures. I have laboured to supply, in some measure, these deficiencies, as I found them recurring in my practice; and now venture to lay before my professional brethren these results of my study and experience, in the hope that their embodiment in a simple and practical form may facilitate their labours in design, and promote the comprehension of a branch, which the growing preference for ironwork in engineering structures invests with well-acknowledged importance.

F. W. S.

47, PARLIAMENT STREET, WESTMINSTER,
January 18th, 1861.

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ERRATA.

For the last two lines of Paragraph 33, Page 34, read—
“ $(N - 1) \times (N - 2) + 6 = 6 \times 5 + 6 = 36$, and the actual strain
upon it is $\frac{1}{2} W \times \frac{36}{49}$.”

In Fig. 21, Plate 3, *for the numbers 0, 1, 3, 6, 10, 15, 21, 28,*
read “0, 2, 6, 12, 20, 30, 42, 56.”

THE STRAINS

ON

STRUCTURES OF IRONWORK.

1. An increasing preference for the employment of ironwork in engineering and architectural structures, is a feature in the history of modern practical science. Since the completion of the Britannia Tube, in particular, the subject has received a rapid development; and such works can now be constructed, not only on a scale of greater magnitude, but, (though the material itself is more costly than any other in general use,) with greater economy than before. Such an apparent anomaly is due, partly to the inherent strength of iron, and partly to its peculiar capacity for being formed into such dimensions and shapes as shall meet the varying requirements of the strains on each portion of the structure. Thus the requisite quantity of the material is supplied in each case, without any being lost to use, and an economy upon the whole is the result.

2. A knowledge of the scientific principles on which such strains are determined, as well as a practical ac-

quaintance with the manufacture of ironwork, are therefore the essential qualifications of the engineer who designs this description of work. Their elucidation is the object of this treatise, and it is therefore proposed briefly to enter upon the investigation of the strains, as they are found in the forms of iron construction in general use.

3. The pressures on loaded beams or girders will first be considered, in which the several parts of the structure are subject to strains of tension and compression, according to their positions in the beam and to the position of the loading.

Commencing with the simplest case, where a beam is supported at each end, and sustains a weight in the centre, as in Fig. 1, the following strains are produced.

The weight is first received by the beam, and is transmitted eventually by it to the piers or points of support. Each of these piers, when the weight is at the centre, sustains a vertical pressure equal to one half of the weight, and (by the law of mechanics, that action and reaction are equal,) impresses upwards in return upon the end of the beam resting upon it, a vertical strain of the like amount which is transmitted through the beam.

Independently of these vertical pressures, there are horizontal strains induced at the centre which act along the top and bottom of the beam.* The tendency of the weight is to deflect and break the beam in the middle by compressing or crushing the horizontal fibres in the top portion, and by extending or tearing asunder the fibres at the bottom. The compressive strain at top,

* These pressures are usually provided against in iron girders by placing a suitable quantity of metal in the shape of flanges along the top and bottom of the girder.

and the tensive at bottom are equal, and by considering each half of the beam as a lever, are found equivalent to one half the central weight, increased by the leverage due to half the length of the beam divided by its depth. Thus if W be the weight, L the length or span, and D the depth, the strain on the top or bottom flange will amount to $\frac{1}{2}W \times \frac{\frac{1}{2}L}{D} = \frac{W \times L}{4 D}$ at the centre ;

and when the girder is of uniform depth as in Fig. 1, these strains diminish regularly and gradually from the centre to the end, where they disappear.

It follows conversely from this rule, that if the depth were diminished regularly from the centre to the ends, as shewn by the dotted lines in Fig. 1, the strains on the top and bottom flanges would be uniform throughout with a central load.

4. When the weight is placed at unequal distances between the piers, the portions of it sustained by each pier are in the inverse ratio of the distances of the weight from the piers. Thus, if the weight be placed at one-fourth of the span, the further pier from the weight will sustain one-fourth of it, and the nearer pier three-fourths ; and the vertical pressures in the girder on each side of the weight will be in the like proportion.

The horizontal strains at top and bottom, as described in the last paragraph, are found by multiplying the portion of the weight transmitted to one of the piers, by the distance from the weight to that pier, and dividing the product by the depth of the beam.

5. When a beam, as in Fig. 2, is uniformly loaded throughout its length, each pier sustains a pressure equal to one-half of the total weight, and the end of the beam receives by reaction, as described in paragraph 3,

a vertical pressure of the same amount; then if W be the total weight of the loaded beam, L its length, and D its depth, the following expressions will give the strains. The vertical pressure commences at the centre, and increases or accumulates regularly to each end, where it is $\frac{1}{2}W$, and the horizontal strains at top and bottom at the centre of the beam are equal to $\frac{W \times L}{8 D}$. Comparing this expression with that in paragraph 3, it follows that a distributed load causes only half the horizontal strains at the centre produced by the same loading if collected at the middle of the beam. When the depth of the girder is uniform, the horizontal strains diminish from the centre to the ends, where they disappear, but the proportion of diminution is not regular, as in the case of a central load. The ratio of the strain at the centre to the strain at any other point, is as the square of half the length of the beam to the rectangle or multiple of the segments into which the beam is divided at that point. In this way it is found, that if the beam be divided into ten equal parts, and the strain at the centre called 1, the strain at the other parts in succession counting from the centre will be .96, .84, .64, .36, and 0 at the end; and, if the actual strain at the centre be multiplied by these decimal numbers, it will give the strain at those points. It follows, conversely from this rule, that if the depth were varied in the same proportions, as shown by the dotted lines in Fig. 2, the strains on the top and bottom flanges would be uniform throughout with an equally-distributed load. It should also be observed, that when the distributed loading extends over a portion only of the structure, the centre is subject to an increase of vertical pressure which

reaches a maximum when one-half of the beam is so loaded; the central vertical pressure is then equal to one-eighth the weight of the fully-loaded beam. This often occurs in practice, as, for instance, in the case of a railway bridge which is variably loaded during the passage of trains.

FRAMED GIRDERS.

6. The simplest form of framed girder for the purposes of calculation, though not the best for practical use, is that with the top and bottom flanges parallel and connected by single triangular bracing, as shown in Fig. 3. If such a girder be loaded either at the centre, or at each of the points of meeting of the braces,* the following strains will be called into action, viz. :—

- 1st. A strain of compression on the top horizontal member or flange.
- 2nd. A strain of tension on the lower horizontal flange.
- 3rd. Vertical strains of alternate compression and extension on the braces, which convey the load to the piers.

Supposing in the first instance, a weight W , Fig. 3, placed at the centre, then one-half of it is transmitted by the framing to each pier alike, and the effects upon the structure are as follows :

The weight $\frac{1}{2}W$ transmitted to pier 1, first bears upon the parts next it k and a , and as k is horizontal and unsuited to sustain vertical loading, the weight

* A distributed load over a framed girder of any description is usually dealt with for convenience of calculation, as if collected in equal portions at the points of meeting of the braces.

$\frac{1}{2}W$ is supported by the brace a , upon which it induces a tension equal to $\frac{1}{2}W$ increased in the proportion of the length of the brace a to its vertical depth from W to g . The tension in a then pulls upon the parts g and b to which a is connected, and causes strains of compression upon them. The compression in the bay g is met and sustained by an equal and opposite strain upon it, produced in like manner by the second half of W while being conveyed to the further pier 2,* and the compression in b acting downwards, causes tension on c and k , the latter being met as at top by an equal and opposite tension from the other side of the centre. In like manner the tension on c causes compression on d and on h , the compression on d causes tension on e and on l , the tension on e causes compression on i and on f , and lastly, f causes tension on m , and transmits the weight $\frac{1}{2}W$ to its final support on the pier.

7. In this process, the braces a , c , and e , cause strains of compression on the top flange, acting in each case from the point of junction of the flange and brace up to the centre of the girder at g ; so that the central bay g receives three distinct compressive strains, the second bay h two strains, and the end bay i one. In like manner the braces b , d , and f , which cause tension on the bottom flange, induce three, two, and one, distinct strains, respectively on the bays k , l , and m ; and all these strains, both tensile and compressive, are met by equal and opposite strains on the other side of the centre. The strains upon the braces are equal through-

* It may be useful to younger students to state, that these two strains, which re-act upon each other, cause a single pressure only upon the horizontal part g ; so also with the bottom central bays.

out in a regular figure, and are tensive and compressive alternately.

8. If the vertical depth on the diagram be supposed to represent $\frac{1}{2}W$, the lengths of the bays and braces will represent in proportion the amounts of their own respective strains, which may be arrived at geometrically, thus:—Let Fig. 4 represent the half of Fig. 3, and make $s t = \frac{1}{2}W$ by a scale of equal parts, draw the parallels $v x$ (through t) and $x y$; then, measuring by the same scale, $s x$ gives the tension on the brace a , and by the resolution of forces, $s v$ and $s y$ give the compressions on the next brace and on the top flange. The strains on the bays g , h , and i , Fig. 3, will, as explained in the last paragraph, be respectively, thrice, twice and once, that indicated by $s y$ in Fig. 4.

In the last bay m , of the longer flange, however, the tensive strain is but one-half that on the other bays. This may be seen by supposing the vertical support of the pier replaced by an equivalent weight hung over a pulley as shown in Fig. 4; when the parallelogram of pressures resulting from the strain on the last brace which puts ca and cd into tension, is represented by $abcd$, and the strain on the last bay by cd , or half that of the other bays. The terms of progression of the strains on the bays of the longer flange, therefore, instead of 1, 2, 3, as above-mentioned, will be $\frac{1}{2}$, $1\frac{1}{2}$, and $2\frac{1}{2}$.

9. When the weight is at unequal distances between the piers, the portions of W supported by each pier are in the inverse ratio of the distances of the piers from the weight, which ratio may be conveniently expressed in terms of the number of bays. Thus, referring to Fig. 3, if W be placed at the lower end of the braces

c and b , which is two bays from pier 1, and four bays from pier 2, there being six bays altogether in the structure, the pier 1 will support four-sixths, and the pier 2 two-sixths of the weight. Again, if W be placed at top, at the intersection of the braces d and e , which is one and a half bay from pier 1, and four and a half bays from pier 2, the pier 1 will support $4\frac{1}{2}$ sixths or nine-twelfths of the weight, and the pier 2 three-twelfths. These respective portions of the weight will be conveyed to the piers by the framing, and cause strains upon each part of the structure, in the manner described in paragraphs 6, 7, and 8, for a load at the centre.

10. It is shown in Fig. 3, that a weight at bottom causes tension in the two braces connected with it. A weight at top, on the contrary, causes compression in the two adjoining braces, and the strains alternate in each case upon the braces from the weight to each pier.

11. When weights are placed either at several or at all the points of intersection of the braces, each weight acts separately throughout the girder as described in the five preceding paragraphs, and the sum of their separate effects will be the total effect on each part of the structure. The weights always produce compression on the top flange and tension on the bottom, so that the strains on these members are invariable in their nature, and are greatest when the structure is loaded at every point. But in the braces the combined effect is different, as the strains upon them vary in nature as well as in amount, according to the position of the loading. For example, it will be seen from Fig. 3, that a weight suspended from the bottom causes tension in the two adjoining braces, and in each brace inclined in

the same direction to them from where the weight is placed to each pier respectively ; it also causes compression in the braces inclined in the opposite direction. Now a little consideration will show that if the two points next pier 1 Fig. 3, were left unloaded, and weights placed at the other three points of intersection of the braces with the bottom flange, there would be tensive strains only caused in the central brace a ; and on the other hand, if the two points next pier 1 were loaded, and none of the others, there would be compressive strains only caused in the same central brace. Therefore, in designing a structure of this nature, subject to partial or movable loading, as occurs, for instance, during the passage of a railway-train, the brace a must be made capable of resisting separately both compressive and tensive strains to these amounts, so as to have the full strength required in every condition of loading to which the structure is liable. The end braces, however, are subject to strains of one nature only, which are compressive in the form shown in Figs. 3, 5, and 7, where the last braces bear upon the piers ; and tensive in the form shown in Figs. 6 and 8.

The strain on the end braces, when the girder is loaded at all the points, is equal to half the sum of the weights (or half the entire weight of the structure), increased in the proportion of the length of the inclined brace to its vertical depth.

12. The above general remarks will render the subject sufficiently intelligible to the practical engineer ; and it is now proposed to give the necessary formulæ by which the strains on each part of the structure can be calculated in the cases occurring in practice.

13. In the case of girders with triangular bracing

loaded at the centre, as in Fig. 3, the strain on the top and bottom flanges at the central bay or pair of bays is found thus :

$$\text{Strain} = \frac{W \times L}{4 D}$$

in terms of the central weight, length, and depth, as before.* The strains in this case increase regularly and in arithmetical progression from the first bays next the piers to the central bays, the terms of this progression being 2, 4, 6, 8, &c., for the shorter flange (such as the upper flange in Fig. 3), and 1, 3, 5, 7, &c., for the longer flange (such as the lower flange in Fig. 3), which are equivalent terms of series to those mentioned in paragraph 8, but given in whole numbers for convenience.

Therefore the actual central strain $\frac{W L}{4 D}$ being found, then if a be called the term of the above progression applicable to the central bay, and b the term of the progression applicable to any other bay, the central strain divided by a , and multiplied by b , will give the actual strain on that other bay. Thus if the central weight in Fig. 3 be 12 tons, the length between the supports 60 feet, and the vertical depth 6 feet, then the strain on the central bays $\frac{W L}{4 D} = \frac{12 \times 60}{4 \times 6} = 30$ tons. The strains on the other bays are calculated thus :—

Proportional numbers for bays in upper flange 2, 4, 6, the number applicable to central bay being 6.

Ditto for bays in lower flange 1, 3, 5, the number applicable to central bay being 5.

* The strain on the central bays of the longer flange is in reality somewhat less than that on the central bay of the shorter, as explained in the latter part of Paragraph 8; the difference, however, is considered immaterial in practice. *

Strain on first upper bay . . .	$\frac{30}{6} \times 2 = 10$ tons.
Do. on second bay . . .	$\frac{30}{6} \times 4 = 20$ tons.
Do. on third or central bay . .	$\frac{30}{6} \times 6 = 30$ tons.
Strain on first lower bay . . .	$\frac{30}{5} \times 1 = 6$ tons.
Do. on second do. . .	$\frac{30}{5} \times 3 = 18$ tons.
Do. on third do. . .	$\frac{30}{5} \times 5 = 30$ tons.

The strains on the braces are in compression and tension alternately, but uniform throughout in amount, and are equal to half the central weight W increased in the proportion which the length of the inclined brace bears to its vertical depth, which may be measured by a scale of equal parts on a diagram of the girder as explained in paragraph 8.

14. In the case of girders with triangular bracing loaded by equal weights placed at top or bottom, at all the points where the braces meet the flange, the strains on the various parts are found as follows:

The strain on the central bay or pair of bays in the top and bottom flanges, is equal to $\frac{W L}{8 D}$ in terms of the total weight, length, and depth of the structure. For the other bays, if N be the total number of bays in the longer horizontal member (such as the lower flange in Fig. 3), and n the number of any particular bay counting from the end of the longer flange towards the centre, being 1, 2, 3, 4, &c., in succession, the formula $2n (N + 1 - n) - (N + 1)$ will give a series of numbers proportional to the strain on each bay from the pier to the centre. Then if a represent the number so found for the central bay, and b the corresponding number for any

other bay, the strain on that bay will be equal to the central strain $\left(\frac{W L}{8 D}\right)$ divided by a and multiplied by b .

For the shorter horizontal flange, such as the upper flange in Fig. 3, the same rule will apply, excepting that n must be represented by the series $1\frac{1}{2}$, $2\frac{1}{2}$, $3\frac{1}{2}$, $4\frac{1}{2}$, &c., in succession from the pier to the centre, being in fact the horizontal distance from the point of support, expressed in the number of bays.

Thus, if the structure represented in Fig. 3 be 60 feet in length, 6 feet in depth, and loaded at each point on top with a weight of 8 tons, giving 48 tons total load, then the strain on central bays g and k being $\frac{W L}{8 D}$ is equal to $\frac{48 \times 60}{8 \times 6} = 60$ tons.

The terms of strains on the bays of Fig. 2, found by the formula $2 n (N + 1 - n) - (N + 1)$ where N is equal to 6, are as follow:—

Proportional Numbers for the Longer (lower) Flange.

For the first bay . . $(2 \times 1) \times (6 + 1 - 1) - (6 + 1) = 5$.

For the second bay . $(2 \times 2) \times (6 + 1 - 2) - (6 + 1) = 13$.

For the third bay . . $(2 \times 3) \times (6 + 1 - 3) - (6 + 1) = 17$.

The actual strain on centre bay being 60 tons, it follows from the formula $\frac{60}{a} \times b$ given above, that—

Actual strain on first bay, next pier, $= \frac{60}{17} \times 5 = 17.65$ tons.

Do. on second bay . . . $= \frac{60}{17} \times 13 = 45.89$ tons.

Do. on third bay . . . $\frac{60}{17} \times 17 = 60$ tons.

Proportional Numbers for the Shorter (Upper) Flange.

For the first bay . . $(2 \times 1\frac{1}{2}) \times (6 + 1 - 1\frac{1}{2}) - (6 + 1) = 9.5$.

For the second bay . $(2 \times 2\frac{1}{2}) \times (6 + 1 - 2\frac{1}{2}) - (6 + 1) = 15.5$.

For the third bay . . $(2 \times 3\frac{1}{2}) \times (6 + 1 - 3\frac{1}{2}) - (6 + 1) = 17.5$.

Actual strain on first bay	$\frac{60}{17.5} \times 9.5 = 32.58$ tons.
Do. on second bay	$\frac{60}{17.5} \times 15.5 = 53.16$ tons.
Do. on third bay	$\frac{60}{17.5} \times 17.5 = 60$ tons.

15. In the same description of girder, viz., with triangular bracing and distributed loading, the strains on the braces are found as follows.

The strain on the brace next the pier, as already stated in paragraph 11, will be either wholly compressive, (as in Fig. 4,) or wholly tensive, (as in Fig. 4, if inverted and supported from above or below at the same point *c*.) In either case, as before mentioned, the strain on the end brace will equal half the total weight of the loaded structure increased as the length of the brace to its vertical depth.

The nature of the strains in the other braces is easily seen from the character of the girder and the position of the loading. Thus in Fig. 3, if all the weights be on top, the last brace *f* is strained wholly in compression, and the next brace *e* is subject to a small compressive strain from the portion of the last weight at the apex of *f* and *e* (in this case one-twelfth), which is transmitted to pier 2, and to a tensive strain from all the other weights; also the brace *d*, which is in mutual connection with *e*, will receive as much compressive strain as *e* receives tensive, and as much tensive strain as *e* receives compressive. So again with the pair of braces *c* and *b*, which similarly and mutually receive strains of equal amount, but of opposite character, the compression on *c* being equal to the tension on *b*, and the compression on *b* being equal to the tension on *c*; and so with each succeeding pair of braces to the centre of the girder. If all the weights be at the bottom,

however, the last weight is at the lower end of the braces *e* and *d*, and *f* receives no weight but what is transmitted through *e*; therefore *e* and *f* are equally strained, *e* being wholly in tension and *f* wholly in compression, and the strain on each being that caused by half the weight of the structure, increased in the ratio of the length of the brace to its depth. The following pairs of braces from these, viz. *d* and *c*, *b* and *a*, &c., will be mutually subject to strains of equal amount, but of opposite character, as before explained.

Regard being thus had to the position of the loading and to the nature of the strains which result from it, the following formulæ give a series of numbers expressing the proportionate strains on each pair of braces as above described, in terms of the number of bays *N* contained in the longer horizontal flange.

	Strains of compression or tension.	Strains of tension or compression.
For the brace or pair of braces } as the case may be, next the pier. }	N^2	none
For the second pair of braces . .	1	$(N-1)^2$
For the third pair of braces . .	$(N-2)^2$	4
For the fourth pair of braces . .	9	$(N-3)^2$
For the fifth pair of braces . .	$(N-4)^2$	16
For the sixth pair of braces . .	25	$(N-5)^2$
For the seventh pair of braces . .	$(N-6)^2$	36
And so on.		

To illustrate this subject with more distinctness, and with convenience for practical use, the diagrams in Figs. 5, 6, 7, and 8 are given in the plates, which exhibit the distribution of these strains in the cases commonly occurring in practice.

In this manner a series of numbers is found, representing proportionate values of the strains on each brace, in terms of the number of bays in the longer flange. The actual strains can then be found thus. That on the

end brace, as before mentioned, is equal to half the weight of the loaded structure, increased in the ratio of the length to the depth of the brace. Then if a be this actual strain on the end brace, b the proportionate number on the end brace, and c the proportionate number on any other brace, as found by the above formulæ, the expression $\frac{a}{b} \times c$ will give the actual strain on that other brace.

16. It may here be appropriate to refer again to what was stated in the latter part of paragraph 11, respecting the necessity of making the braces capable of resisting separately the full amounts of tensive and compressive strains to which they may be subjected by variable loading. Should the load however, be not liable to increase or diminution, or should the strains be caused by the permanent weight of the structure itself without extraneous loading, the difference only of the compressive and tensive strains on any given brace need be provided for, when apportioning the strength of the material of which that brace is composed. Thus if the above formulæ give nine tons of compressive and four tons of tensive strain on a brace, in a structure not liable to variable loading, the actual strain to be provided against would be only five tons of compression. It is only, however, in large structures that the distinction between the two cases is appreciable in practice; and its effect is never felt in the end braces, which receive the greatest stress, and are only subject to strains of one nature.

17. It may be observed with reference to Fig. 3, that if the loading be placed at top, there are six weighted points, but if at bottom, only five, for the same girder. The difference in such cases is not material in practice

and the full weight of the loaded structure may be included in the sum of the weights, whether five or six: in which case a weight on one of the five loaded points would be of course somewhat greater than a weight upon one of the six, though their total amount would be the same.

CANTILEVERS.

18. The strains on a loaded cantilever as in Fig. 9, are similar to those on a loaded girder as in Fig. 1, excepting that on the horizontal members they are reversed, the top flange now sustaining a strain of tension and the bottom of compression. This is evident by supposing a weight placed at *c*, Fig. 9, which in deflecting or forcing down the end *cd*, causes a compressive or crushing effect from *d* to *b*, and an equal tensive effect from *c* to *a*. These strains being induced by the end weight are equal to that weight modified by the leverage or the proportion of length to depth of the cantilever; and when the depth is uniform, as in Fig. 9, they commence at the end *cd*, and increase regularly to the place of support at *ab*, where they are equal to $W \times \frac{L}{D}$ in terms of the end weight, and the length and depth of the cantilever. There is also a vertical pressure equal to *W* upon the pier at the point *b*, with an equal reaction throughout the cantilever itself, as explained in paragraph 3 in reference to the case of a loaded girder.

It follows conversely from the above statement, that if a cantilever loaded at the end be formed, with the depth regularly increasing as in Fig. 10, the strains on the top and bottom flanges will be uniform throughout.

19. In the case of a cantilever with equally-distributed loading, the effect at the place of support is, by

a law of mechanics, equal to that caused by the same loading collected midway upon the length of the lever. Therefore the greatest horizontal strains upon the top and bottom flanges, (which are produced as above explained, at the places of support *a* and *b*, Fig. 9,) are equal to $W \times \frac{\frac{1}{2}L}{D}$ or to $\frac{1}{2} W \times \frac{L}{D}$, in terms of the total weight, and the length and depth of the cantilever; being the same amount of strain at *a b*, that would be caused by half the distributed load placed at the extremity *c d*. Supposing the depth to be uniform, the rate of increase of the horizontal strains from the end to the place of support is not regular, as in the case of a cantilever loaded at the end, but is in proportion to the squares of the distances from the end. In this way it is found, that if such a cantilever be divided into five equal parts, and the horizontal strain at the place of support be called 1, the strains at the other points will be respectively .64, .36, .16, .04, and .0 at the end;* and if the actual strain at the place of support be multiplied by these decimal numbers, it will give the strain at those points.

It follows conversely from the above statement, that if the depth be diminished in these proportions from the place of support to the end, as in Fig. 11, the strains on the top and bottom flanges will be uniform throughout with a distributed load.

The vertical pressure on the place of support at *b* is equivalent to the total weight *W*, with an equal pressure reacting upon the part *a b* of the cantilever above it this pressure diminishes regularly through the cantilever from thence to the end at *c d*, where it disappears.

* These numbers are proportional to the series of squares applicable in this case, viz. 25, 16, 9, 4, 1, and 0, at the end.

CANTILEVERS WITH TRIANGULAR BRACING.

20. From the general explanations given in the two preceding paragraphs, the strains upon framings of this kind, such as shown in Figs. 12 and 13, are easily deduced. When they are loaded only at the extremity c , the strains on the braces are equal in quantity throughout, and are equivalent to the end weight increased in the proportion of the length of the inclined brace to its vertical depth, which may be found by a scale of equal parts, as described in paragraph 8. The strain on the bay of the longer flange next the place of support, where it is greatest, is equal to $W \times \frac{L}{D}$, in terms of the end weight, and the length and depth of the cantilever. The strains on the end bay next the weight at c , are equal to the greatest strain as found above, divided by the number of bays; and the strains on the other bays commencing from c are respectively twice, thrice, four times, &c., the strain on the end bay next c , increasing in regular proportion from c to a . Upon the shorter flange db , however, the strain on the end bay c , as found above, must be multiplied by $1\frac{1}{2}$, $2\frac{1}{2}$, $3\frac{1}{2}$, &c., to give the strains on its bays in succession, commencing from the end d .

21. When braced cantilevers, such as Figs. 12 and 13, are loaded at each point of intersection of the braces from c to a , the strains on the pair of braces next to c are equivalent to one of the weights increased in the proportion of the length of a brace to its vertical depth. The strain on the second pair of braces is double this amount, as they receive the second weight in addition, for transmission to the place of support. The strain on the third pair of braces is treble, and so

on in proportion, with a regular increase from c to the place of support at $a b$.

The horizontal strains on the flanges arising from an uniform loading on Figs. 12 and 13, are thus found.

Each weight, at the intersection of the braces, is considered to act separately, as described in the last paragraph, from the point at which it is placed to the point of support; and the addition of these separate effects gives the total effect on each bay. Commencing in this manner with the end bays at c and d , these effects on the several bays of the longer flange $a c$ are found proportionate to the numbers 1, 3, 6, 10, 15, 21, &c. (being the additions of the terms of the series 1, 2, 3, 4, 5, 6, &c.); and the effects on the bays of the shorter flange $d b$, are proportionate to the numbers $1\frac{1}{2}$, 4, $7\frac{1}{2}$, 12, $17\frac{1}{2}$, &c. (being the additions of the terms of the series, $1\frac{1}{2}$, $2\frac{1}{2}$, $3\frac{1}{2}$, $4\frac{1}{2}$, $5\frac{1}{2}$, &c.).*

The actual strain on the end bay at c is $\frac{W \times L}{D \times N}$ as given in the last paragraph, in terms of the single weight at the extremity c , of the number of bays in the longer flange, and of the length and depth of the cantilever; and if this strain be multiplied by the above proportionate numbers, the strains on the other bays in succession to the place of support will be found.

CONTINUOUS GIRDERS.

22. In a girder of this description, extending over two spans, as shown in Figs. 14 and 15, the portion next the abutment for the length of three-fourths of

* The series 1, 2, 3, &c., as applied to the longer flange, and the series $1\frac{1}{2}$, $2\frac{1}{2}$, $3\frac{1}{2}$, &c., as applied to the shorter, are in fact the horizontal distances from the extremity c , expressed in the number of bays.

one span, should be treated as if a detached and independent girder with the loading which is due to its length, and the strains upon each of its parts should be calculated accordingly.

The remaining one-fourth should be treated as a cantilever projecting from the central pier, and supposed also to be detached and independent. The strains induced upon it are as follow. One-half of the total weight of the supposed separate girder above-mentioned (or the weight of three-eighths of one span), is taken as placed at the extremity of the cantilever at *a*, Figs. 14 and 15, and the strains caused by it are calculated in the manner explained in paragraph 20. In addition, there are the strains from the loading upon the cantilever itself, whereof the mode of calculation is given in the paragraph preceding; and the sum of these double effects gives the total strain upon each portion of the supposed cantilever.

If the distance of three-fourths of the span from the abutment fall intermediately on a bay, it will be better, for convenience of calculation, and for security of construction, to include that bay in the separate lengths both of the girder and cantilever as supposed.

The point *a*, Figs. 14 and 15, where the supposed girder and cantilever unite, is usually called the point of contrary flexure, because a loaded continuous girder assumes a curved outline which is concave from the abutment to this point, and convex from thence to the central pier, as shown in Fig. 16.

It is also to be especially noted that the horizontal strains over the pier become tensive in the upper flange and compressive in the lower between the points of contrary flexure, as shown in Fig. 16; the portion treated

as a cantilever having its strains reversed as compared with those of a girder, as was shown in paragraph 18 and in Figs. 12 and 13.

UNEQUAL LOADING OF CONTINUOUS GIRDERS.

23. When one of the spans is more heavily loaded than the other, (which takes place, for example, in the case of a railway train when occupying one span only during its passage of the bridge,) the position of the point of contrary flexure is brought nearer to the central pier, and the portion of the structure next the abutment must be calculated as a girder of greater length than three-fourths of the span. The following formula will give its length in this case:—

Let W be the weight of the heavier span;

W^1 be the weight of the lighter span;

And L the length of one span;

Then $\frac{7W - W^1}{8W} \times L = \text{Distance from abutment to point of contrary flexure.}$

This distance should be taken in practice as the length of the girder portion next the abutment, of which the strength should be calculated accordingly. But the length of the cantilever portion must still be estimated as one-fourth of the span, since it becomes so when both spans are fully loaded. In fact, the point of contrary flexure is varied in its position by unequal loading, and both girder and cantilever must be made suitable for the extreme variation in each case.

24. In the case of a continuous girder of three or more spans uniformly loaded, the distance of the point of contrary flexure is altered in the first opening to four-fifths of the span instead of three-fourths; and in the middle span or spans, the length of the cantilever portion is increased from one-fourth or twenty-five

hundredths to about twenty-eight hundredths of the span :—the remaining portion of forty-four hundredths of the central span being treated as a detached girder as before described. In other respects the girders are treated similarly to those over two openings, as above described.

25. It should be noted that when the last braces over either the abutments or piers are in tension, as shown in Fig. 15, a vertical strut or some other arrangement becomes necessary, for transmitting the weights borne by the end ties to the point of support on the abutment or pier. In the case where there are two spans in the bridge, as in Fig. 15, the vertical strut shown at the abutment is loaded with three-eighths of the weight of one span; and the vertical strut shown at the central pier with five-eighths of the weight of both spans, which is equivalent to the weight of one and a quarter span. If the girder extend over three spans, the vertical strut at the abutments would receive two-fifths the weight of one span; and the vertical strut at the pier would receive three-fifths the weight of the first span, and one-half the weight of the second span, equal together to the weight of one and one-tenth span. Lastly, when the girder covers more than three spans, the abutments and first piers are (with some slight modifications immaterial in practice) loaded as last described; and the other or intermediate piers sustain the weight of one span.

PARALLEL LATTICE GIRDERS.

26. These girders, which are excellent constructions, consist of a double, treble, or other multiple combination of the girder with single triangulation which has just been treated of. Thus Fig. 17 represents a double combi-

nation of the girder shown in Fig. 3; and in this case the strains are calculated separately for each single system of bracing according to the rules already given; the weights at the intersection of the flange and braces being half of those on a girder of single triangulation, as there are double the number of points to be loaded. The strains upon the braces are thus found separately, one-half their number from each system; and the sums of the several strains upon the bays of the horizontal flanges give the total amounts of strain upon those bays, as may be seen from the diagram Fig. 17, where a , b , and c , represent the strains produced on the first system, and d , e , and f , the strains produced on the second.

The same principles apply to a combination of three or more sets of single triangular bracing; the weight at each intersection of the flange and braces being diminished proportionately (with a given total load) as the sets of triangles are increased.

PARALLEL GIRDERS WITH VERTICAL STRUTS.

27. The most advantageous mode of constructing girders of the forms shown in Figs. 18 to 21, is to make the vertical braces struts, and the inclined braces ties. They will then, if properly designed, contain a good combination of bracing, and have much advantage as regards the struts, which are shorter than the inclined ones in the triangular system, and do not undergo an increase of pressure from being placed in a slanting position. This advantage is the more felt in struts, as their liability to bend or buckle increases not only in proportion to the load upon them, but to the square of their length; whereas the nature of the strain upon a tie tends to keep it in a straight

position, and its length is therefore comparatively immaterial.

28. The author has seen several girders on this principle erroneously designed, both for buildings and for large bridges subject to variable loading. It is proposed therefore to investigate the effects of each weight upon the framing, so that the requirements of the structure may be understood.

For a girder of this description with a central weight on top, as in Fig. 18, the bracing therein shown is required, and the following effects take place. The whole weight is sustained in the first instance by the central strut *a*, from whence one-half of it is received and transmitted towards each pier by the ties *b* and *c*, which ties undergo strains of tension equal to one-half of the weight increased in the proportion of their length to their vertical depth. Tracing the effect on one side of the centre, the tension on *b* impresses, by the resolution of forces, compressive strains on *d* and on *e*. As *d* is horizontal, and therefore unable to sustain a vertical load, the strut *e* sustains the weight $\frac{1}{2}W$; and carrying it onwards towards its support on the pier, impresses tensile strains on *f* and on *h*. The tie *h* receives the vertical load, and causes compressive strains on *g* and on *i*, and the strut *i* transmits finally the weight $\frac{1}{2}W$ to the pier. The same effects take place on the side of pier 2.

The exact amount of these strains may be measured thus. Referring to Fig. 18, suppose $W = 6$ tons, then make $sv = \frac{1}{2}W$ or 3 by a scale of equal parts, and draw the parallels *vt* and *tr*; then *ts* measured by the same scale will give the tension on the tie brace, and *rs* the compression on the upper bay, which acts from *s* up to the centre at *W*. The

same strains are repeated at y , the compression measured by the distance $y x$ also acting from y up to the centre; it follows therefore that the bay next the centre receives two strains, and that next the pier one, which are met by equal and opposite pressures on the other side of the centre.* The compression on the central strut a is $=W$ or 6 tons, and on each of the other struts $\frac{1}{2}W$; but if the weight were placed at bottom, the central strut would not be strained.

29. It will be easily seen from the foregoing, that the chief features to be attended to in designing a structure of this nature, are that a load placed at any point upon it must be conveyed in certain proportions to the two points of support; and that the bracing which performs this duty should be made suitable for it. Every single weight, therefore, in the construction now under consideration, requires tie braces extending from the load in the direction of both piers, to fulfil the requirements of the case.

30. The construction of the girder shown in Fig. 18 is unsuited for any other than a central load. If a weight be placed at unequal distances from the piers, the tie braces should be disposed as shown in Fig. 19. In the girder shown in this figure two-thirds of the weight is conveyed to the nearer, and one-third to the further pier, and the strength of the braces should be provided for accordingly.

31. If a girder of the same description be loaded at two intermediate points, it should be constructed as in Fig. 20, in order to transmit the loading properly to

* If there were three bays on each side of the centre, the proportion of the horizontal strains upon them would be as 1, 2, 3, and so on for any number.

the piers as described in paragraph 29. Girders on this principle have been extensively used in the Crystal Palace Building of 1851, and in the other Exhibition buildings to which it gave rise.

32. If there be four or more bays in the girder, with weights at each point, the double diagonals should be furnished to every bay except the last; and they are generally inserted in practice in these also, to increase the stiffness and preserve the symmetry of the girder.

33. It now remains to give formulas in a convenient shape for practical use, by which the strains on these girders may be found.

The strains on the various parts of a girder with central loading, such as shown in Fig. 18, have been fully explained in paragraph 28.

For girders of this kind uniformly loaded, the diagram shown in Fig. 21 gives a series of comparative numbers for the strains throughout the structure, in terms of the number of bays N . To these the following rules may be applied in order to find the actual strains; and in making the calculation in practice, it is convenient to fill in upon a diagram of the intended structure, the numbers found from the formulæ given in Fig. 21, which are called the proportional numbers.

CALCULATION FOR THE TOP AND BOTTOM FLANGES.

Referring to Fig. 21; the actual horizontal strain on the central bay or pair of bays is equivalent to $\frac{W L}{8 D}$, in terms of the total weight, length, and depth of the loaded structure. Then if a be the proportional number applicable to the central bay or pair of bays, and b the proportional number applicable to any other bay,

the actual central strain multiplied by $\frac{b}{a}$ gives the actual strain on that other bay.

CALCULATION FOR THE DIAGONAL TIE BRACES.

The actual strain in the last diagonal, of which the proportional number is $N \times (N - 1)$, is equal to half the weight of the loaded structure increased in the proportion of the length of the tie to its vertical depth. Then if the proportional number, applicable to the end tie brace be called c , and the proportional number applicable to any other tie brace be called d , the actual tension in the end tie brace multiplied by d and divided by c gives the actual tension in that other brace.

CALCULATION FOR THE VERTICAL STRUTS.

The proportional numbers for the vertical struts are found as follows:—If the load be on top the number in terms of N for the first strut over the pier will be N^2 , and for any other strut it will be the sum of the proportional numbers upon the two diagonals meeting at the bottom of that strut. If the load be at bottom, it will be in each case the same number less N ; and these numbers may be easily filled in upon the diagram of the intended structure, after the proportional numbers for the tie braces are put on it as above described. The actual compression upon the struts may then be found thus:—The actual compression upon the end strut is equal to half the weight of the loaded structure, or $\frac{1}{2} W$; and if e be the proportionate number applicable to the last strut, and f the number applicable to any other strut, the actual strain on that strut will be equal

to $\frac{1}{2} W \times \frac{f}{c}$. For example, if there be seven bays in a girder loaded on top, of the construction shown in Fig. 21, the proportionate number for the last strut is $N^2 = 49$, and the actual strain upon it is $\frac{1}{2} W$; also the proportional number for the third strut is $(N - 1) \times (N - 2) + 3 = 6 \times 5 + 3 = 33$, and the actual strain upon it is $\frac{1}{2} W \times \frac{33}{49}$.

BOW AND STRING BRIDGES.

34. The outline of these structures usually consists of a top or upper flange in the form of the segment of a circle, and of a horizontal bottom flange or tie upon which the roadway is carried. Between these flanges a bracing is inserted, which is generally either triangular or with vertical struts as last described; and examples of this construction are given in Figs. 22, 23, 24, and 25. In this arrangement, the thrust which would be exercised by an arch alone on the abutments, is evidently received by the lower flange or tie, and the abutments are required only to be capable of sustaining the vertical weight of the structure.

35. The compressive strain on the upper flange or bow, and the tensive strain on the lower flange or string, are equal to each other, and are uniform throughout.* The amount of this strain on a structure with distributed loading is equal to $\frac{WL}{8D}$ in terms

* Correctly speaking, this property belongs to a different curve from a circular arc, but when the rise of the latter does not exceed about one-fourth of the span, the difference is scarcely appreciable in practice.

of the total weight and length of the bridge and of its depth at the centre.

36. The calculation of the strains on the braces of a bow and string bridge is more complicated than in the case of parallel girders, owing to the inclinations both of the braces and of the portions of the upper flange to which they unite, being continually varied throughout the structure. The easiest practical method of arriving at the strains on the braces is by ascertaining the distribution of the loading, and applying to a diagram of the structure drawn to scale, a geometrical process as follows.

37. When the bow and string girder is formed with triangular bracing in any convenient number of equal bays, as in Fig. 22, and loaded at bottom, the distribution of the weights is as follows:—If the weight at each loaded point be denoted by the number of bays, which is 6 in Fig. 22, the portions of the several weights transmitted to each pier being in the inverse ratio of their distance or number of bays from the piers, will be as the numbers marked on the lower flange of that figure; thus of the weight 6, supported at the point *r*, five parts are conveyed to pier 1, and one part to pier 2, and so on. Examining, therefore, the weights transmitted by the bracing to pier 2, as received at each point of the upper flange, it appears that the point *b* receives through the brace *rb* the portion of the weight at *r* denoted by the figure 1; the point *c*, receives the same 1, and also 2 of the weight at *s* through the brace *sc*, in all 3: the next point at top receives the same 3 and 3 from *t*, in all 6, and so on; the numbers so found being 1, 3, 6, 10, 15, 21, &c., which are the additions of the series 1, 2, 3, 4, 5, 6, &c.

To find the terms of the compressive strains on the braces inclined from the top towards pier 2, draw the lines bp , cp , &c., which are the resultants of the pressures conveyed onwards from the points b , c , &c., to that pier through the framing as mentioned above. From b set down bx by scale $= 1$ being the number on bp , draw xy horizontally to cut bp and yz parallel to a straight line from b to c , then bz measured by the same scale will give the compression on the brace bs , in terms of N the number of bays. Next from c , set down vertically $cm = 3$, being the number on cp , and proceed as before; and so throughout for the strain on these braces in terms of N .

The actual strains may be found by multiplying the above terms by $\frac{W}{N}$, where W is the weight on one loaded point.

The tensive strains on the remaining braces, or those inclined towards pier 1, are found as follows:—

Commencing at the end next pier 2, let ed set off to scale from e represent the actual strain on the brace ie , as found by the method given above, draw df horizontally, and from f draw also by scale a vertical $fg = W \times \frac{n}{N}$; W being the weight on one loaded point, N the total number of bays, and n the number of bays which e is distant from pier 2. Draw gh horizontally, then eh measured by the same scale gives the actual strain on ef ; and so with the other braces inclined towards pier 1 (Fig 22).

38. When this structure is subject to variable loading, as usually occurs in practice, the braces must be made, in each case, capable of resisting the full amount of strain,

both compressive and tensive, found by the above methods, as such strains, of either nature, are liable to be brought separately upon them. When the structure is uniformly loaded, the braces are in fact free from strain, as the compressive and tensive forces on them are equal to and neutralize each other; in which case the bow and string only are strained, as explained in paragraph 35.

It is also to be remarked, that in this construction the central braces are more heavily strained than those next the piers, being the reverse of that which takes place in a parallel girder with uniform loading, where the pressures on the braces increase from the centre to the ends.

39. When the loading is placed at the top of a bow and string bridge with triangular bracing, as shown in Fig. 23, the first weight at *a* is at the distance of half a bay from pier 1, the second weight at *b* is at the distance of one and a half bay, and so on. If, therefore, the weight at each loaded point be called 6 in terms of the number of bays as described in the paragraph 37, the proportionate numbers showing the parts of the weight transmitted to each pier are $\frac{1}{2}$ and $5\frac{1}{2}$ at the point *a*, $1\frac{1}{2}$ and $4\frac{1}{2}$ at the point *b*, and so on, being in the inverse ratio of the distances or number of bays from the piers; and if these numbers be doubled for greater convenience of calculation, the terms 11 and 1 at *a*, 9 and 3 at *b*, 7 and 5 at *c*, &c., as marked on the upper flange of Fig. 23, are obtained. The numbers marked on the resultant lines (see paragraph 37), drawn out from the upper ends of the braces to pier 2, are 1 at *a*, $1 + 3 = 4$ at *b*, $4 + 5 = 9$ at *c*, and so on; these numbers forming in this case the series of squares 1, 4, 9, 16, 25, &c. The process for finding the strains on the

braces, commencing by setting down to scale the numbers marked on the resultants as above, is the same as described in paragraph 37, excepting that the actual strains so found for the braces must be halved in this case, as the proportionate numbers were doubled to avoid fractional terms. The remarks made in paragraph 38 are also equally applicable.

40. When the bracing is arranged as shown in Figs. 24 and 25, in which the verticals are generally formed to act as struts and the diagonals suitable for tension alone, the strains on the braces may be ascertained in a similar manner. Supposing the weights to be placed at bottom, then the distribution of the loading to each pier, and the numbers to be marked on the several resultants as described in paragraph 37, are shown on Fig. 24. The process of finding the strains is nearly similar and may thus be briefly described for a bottom loading, in which case alone the end verticals are in tension.

Of the first weight 6 at r , Fig. 24, five parts are transmitted to pier 1 through the vertical ra and the portion of the bow from a to pier 1; ra undergoing a strain of tension equal to 5 or $N-1$. The diagonal rb conveys the remainder 1 of the weight towards pier 2, and if rv be made by scale = 1, and vw drawn horizontally, rw measured by the same scale gives the proportionate tension on rb ; and these are the only strains to which ra and rb are subject. Next from b set down $bx = 1$, being the number on the resultant bp ; draw xy horizontally and yz parallel to bc , then the measure of bz by the same scale gives the compression on the vertical bs ; also if sk be made equal to bz and kl drawn horizontally, the measure of sl by the scale gives the tension on sc . Proceeding in order with this process, set down

$cm = 3$, being the number on the resultant cp , and find the proportionate strains on ct and td ; and so on to the end of the structure.

41. The above process gives the tension upon all the diagonals inclined as rb , to which the tension upon those inclined as sa , are respectively equal, and thus the strains are found for all the diagonals in terms of N . The strains on the verticals are more complicated, but may be found thus. The first vertical ar , Fig. 24, is in tension equal to $N - 1$, as explained above. The second vertical bs undergoes a compression measured by bz from the weights between it and pier 1 passing through it to pier 2, and also a second compression by the weights between it and pier 2 passing through it to pier 1. Now the strain from the latter is equal to that already found on du , caused by the weights between du and pier 1 passing through it to pier 2. It follows, therefore, that the addition of the strains found on the verticals bs and du in the manner described in the preceding paragraph gives the total amount of pressure on each of them; and so with any corresponding pair of verticals except the two first, which are in tension. Also when as in this case there is a strut at the centre ct , the pressure upon it is, for the same reason, twice the amount of single strain found as above.

42. The actual strains on the braces are obtained by multiplying the numbers so found in terms of N by $\frac{W}{N}$; W being the weight on one loaded point. The formula for the strains on the bow and string is given in paragraph 35.

43. When the bow and string girder, as shown in Fig. 25, and described in the commencement of paragraph

40, is loaded on the top, the strains on the braces are ascertained as follows. The distribution of the loading to each pier, and the numbers to be marked on the several resultants as described in paragraph 37, are shown on Fig. 25. From a set down by scale $am = 1$, being the number on the resultant ap , draw mn horizontally to cut ap , and no parallel to ab , then ao measured by the same scale gives the amount of compression on ar in terms of N . The strain on the diagonal rb is equal to that on ar increased in the proportion of its inclination to the vertical, and may be measured by the same scale on rv , which is found by making $rw = ao$, and drawing wv horizontally to intersect the prolongation of rb . Proceeding in order, from b set down by scale $bx = 3$, being the number on the resultant bp , draw xy horizontally and yz parallel to bc , and measure bz for the pressure on bs and for the vertical value of the tension on sc ; and so on for the various strains throughout the structure in terms of N . Then with regard to the tension upon the diagonal ties, it will be seen that the strains upon rb , sc , td , and ue will have been found, which will give the strains for all the diagonals, as the tension on td is equal to that on tb , and the tension on ue is equal to that on sa . With regard to the vertical struts, the compression on ar is equal to the measure of am or 1, which is likewise the compression on the strut at e ; also for the reasons given in paragraph 41, the total strains on bs and du in terms of N are each equivalent to the sum of the compressions found for those verticals in the process just described; and the total strain on the centre strut ct is twice the compression so found.

44. The actual strains on the braces are obtained

by the multiplication of these totals by $\frac{W}{N}$; W being the weight on one loaded point. The formula for the strains on the bow and string is the same as given in paragraph 35.

GIRDERS OF A FORM NOT BELONGING TO ANY REGULAR FIGURE.

45. It now remains to give a general method of calculating in detail, the strains which arise in a girder of this description; the principle of which may be applied otherwise as occasion requires. If the girder shown in Fig. 26 be taken as an example, it will suffice to trace the effects of a single weight W placed at one of the loaded points of the framing, as the weights at any or all of the other points may be similarly dealt with.

If, therefore, W , Fig. 26, be called 7 in terms of the number of bays as explained in paragraph 9, there will be 4 parts of it transmitted to pier 1 and 3 parts to pier 2 through the framing, being in the inverse ratio of the distances or number of bays from the weight to the piers. The strains caused by the portion 4 transmitted to pier 1 may thus be ascertained in detail. The weight 4 is first conveyed to the point b through the brace Wb , which undergoes in consequence a strain of tension having 4 as its vertical value. It is then conveyed forward towards pier 1 by the parts of the framing bh and bg , and the resultant of the pressures on these parts is in the line of direction bp from b to the pier. To ascertain the strains in the parts about b , draw accordingly the resultant bp , set down bd by scale = 4 as the vertical weight, and draw de horizontally, and ef parallel to the resultant bp ; then be

measured by the same scale gives the tension on the brace bw , and bf the compression on the bay bc . Proceeding in order with the parts towards the pier, draw dk horizontally from d , then bk having the vertical value of 4, represents the resultant of the compressive strains on bh and bg ; and if ki be drawn parallel to bg , and kl parallel to bh , the distance ib measured by the same scale gives the compression on the bay bh , and the distance bl the compression on bg . To find the tension on gw and gh resulting from the compression on the brace bg , make $gm = bl$, and draw the parallels mo and mn in order to resolve mg into go and gn , which latter, measured by the same scale as before, give the tensile strains on gw and gh .

The weight 4 is thus transmitted to the point h , from whence it is carried forward towards the pier by hv and hr . Proceeding, therefore, as before, draw the resultant hp to the pier, set down $hq = 4$ by scale, draw a horizontal line from q to the resultant and parallels from thence to hv and hr , whence it is seen that the compression on hr is given by hy ; that on hv by hx ; the tension on vr by vj ; and that on vg by vu ; all measured by the same scale as before. The weight 4 being now transmitted to the point r , it remains only to set down by scale $rs = 4$, and to draw st parallel to vp , when the measure of rt by the same scale gives the compression on rp , and the measure of ts the tension on vp .

The bottom flange being horizontal, the total tension on gW is measured by $gn + vu + ts$; that on vg by $vu + ts$; and that on vp by ts ; which will be sufficiently evident from the explanations given in paragraph 7.

The strains caused on the other parts of the structure by the portion of W transmitted to pier 2 can be similarly ascertained, and if the structure be loaded at many or all of the points, the sum of the like strains caused by each weight gives the total effect on each part of the structure.

The weight W above referred to is stated in terms of N the number of bays; and the strains found from it bear the same proportion to the actual strains as N bears to the actual weight.

PLATE GIRDERS.

46. These excellent girders are in very general use, and the calculations of the strains upon them are made as specified in paragraphs 3, 4, and 5.

When heavy weights are to be supported, or when great lateral stiffness and stability on the bearings are required, box girders are usually employed, and the calculation of the strains upon them does not differ in any respect from what is described in the paragraphs above referred to.

47. The precise quantity or scantling of the material apportioned to each part of a structure on which the strains have been ascertained, should be regulated by various conditions. Thus if the loading be liable to sudden changes of imposition and removal, the structure should be made stronger than if subject only to an uniform strain; also the quality of the iron, the greater or less necessity of avoiding deflection, and other circumstances, varying in each case, leave this question in a great degree dependant on the experience and skill of the engineer.

COMPARISON OF DIFFERENT KINDS OF GIRDERS.

48. The strains on the several kinds of girders having thus been described, it is proposed to consider, briefly, the comparative advantages afforded by each.

The plate girder is, except perhaps in appearance, undoubtedly superior to the others. It forms under equal conditions a more rigid, trustworthy, and durable structure; its manufacture is simpler in character, and in small girders it is more economical. When the spans, however, exceed eighty or ninety feet in length, the lattice girder is cheaper, and little inferior in strength; and, as bars can be rolled in greater lengths than plates, the lattices in very deep girders may consist of single pieces only, when plated girders must be jointed horizontally to make up the depth of the beam.

The girders with bracings of single triangulation are less rigid than the others, and are open to the serious objection, that the giving way of any one brace, or fastening, involves the failure of the entire structure. They have some advantages, on the other hand, in cheapness and in portability, as they may be conveyed in small and light pieces to places difficult of access, and erected at little expense. A double system of triangulation, however, with the braces united at their intersections, gives equal advantages in this respect, and forms a better construction with little additional cost.

In short, it may be generally stated—supposing the strains and the iron disposed to meet them to be alike in each case, that the comparative advantages of different systems of girders depend on two circumstances. The first is the continuity and perfection of the part uniting the top and bottom flanges; and in this re-

spect, the plate girder manifestly occupies the first place, the close lattice with the braces frequently intersecting, and thus approaching the plate in continuity, occupies the second, and the unconnected single braces the third. The second circumstance is the mode of fastening the several parts of the girder to each other; this is usually done either by rivetting, or by pins and bolts passing through the pieces at their intersection. The former method is greatly superior; it is universally used in plate girders, and frequently in lattice, though its application to the latter is not in all cases equally convenient. The rivets are more easily applicable where the vertical pressures are diffused, as in the case of plate or close lattice girders; and large bolts are more suitable for the single or double triangle system, where the pressures are collected at fewer points and are greater at each.

PROPORTION OF LENGTH TO DEPTH OF GIRDERS.

49. The depth of a girder is usually made from one-tenth to one-fifteenth of the span. The most economical depth as regards quantity of material, is one-twelfth; and an inch to the foot is an excellent proportion for practice; but the depth must frequently be varied to suit the requirements of each case. It is important, however, to state, that the rigidity of the structure is proportionate to the square of the depth; consequently, though the ultimate strength remains the same, the deflection increases very rapidly as the depth is diminished. Thus, in two girders of equal span and loading, and of the respective depths of one-tenth and one-fifteenth of the span, both constructed of proportionate strength to resist the strains upon them, and both loaded to the point of fracture, there would

be equal weights required to break them down, but the deflection of the one would be fully double the deflection of the other.

CHARACTER OF THE WORK.

50. When economy of time and cost is an object, the framing should be of so plain and simple a character, that ordinary workmen, materials, and tools, may suffice for its execution. If the work be designed in such a manner as to demand materials of unusual size and weight, special machinery, or highly-skilled workmen, it can only be executed by a limited and highly-paid class of persons, and its cost is immensely enhanced. Such a system is, generally speaking, unnecessary and wasteful; and to avoid it, a practical acquaintance with the processes employed in preparing iron for these works, is requisite to those who have the charge of designing them.

The work should consist, as far as possible, of a repetition of the same processes throughout the structure; the materials of ordinary size and of similar dimensions, with the rivets or other fastenings disposed at like places in each. Such work is always preferred by manufacturers, and eagerly sought; and it may be adopted, with extremely few exceptions, even in structures of the very largest class.

IRON ROOFS.

51. The framings generally used for roof principals are simple in their character, and the strains upon them easily ascertained.

In the description of framing shown in Fig. 27, which is usually adopted for roofs of moderate dimen-

sions, the strains on the tie and rafters are greatest at the ends and decrease to the centre, and the strains on the braces are greatest at the centre and decrease to the ends. The loading is supposed to be placed at each intersection of the braces and rafters, and its action on the framing may be thus explained. Referring to the weight W in Fig. 27, which represents a roof with six bays, the portions of W transmitted to each pier are four-sixths to pier 1, which is conveyed directly to the point of support by the rafter WA ; and two-sixths to pier 2, which is first received by WC and WD , and transmitted by them and the vertical tie DC to the point C in the manner already fully described in paragraph 45; whence it is carried by the rafter CB , to the place of support on pier 2. The amount of strain on each part may be measured by drawing a resultant line from W to pier 2, and proceeding as explained in paragraph 45.

52. When the bottom tie in this framing is horizontal, and the structure loaded at every point, a series of numbers may be found in the manner described in the foregoing paragraph, giving the proportionate strains on each part in terms of N , the number of bays into which the two rafters are divided. The formulæ for these numbers are marked upon Fig. 28, and are applicable to any number of bays. The term a as applied to the rafters is equal to $\frac{1}{2} N \times \frac{AB}{BC}$; and the

term b as applied to the tie, equal to $\frac{1}{2} N \times \frac{AC}{BC}$: being the number of bays in one only of the rafters multiplied respectively by the lengths of rafter and tie, and divided by the rise of the roof.

The proportional numbers in terms of N on the

vertical ties form the series $\frac{1}{2} N$, N , $1\frac{1}{2} N$, $2 N$, &c.; but the number for the central vertical tie must be always doubled, as it receives a strain from each side of the roof. The strains on the inclined struts are found by drawing lines from each loaded point on one rafter to the further pier, and marking these lines in succession, with the numbers 1, 3, 6, 10, 15, &c.; the reason of which has been sufficiently explained in paragraph 37. To find the proportional strain on any inclined strut, say the fourth from the end in Fig. 28, set down op by scale equal to 10, the number on the line from o to the further pier, draw pr horizontally to intersect this line, and ts parallel to the rafter; then os measured by the same scale gives the compression on the inclined strut. The strains on the other inclined struts may be found in like manner, setting down vertically by scale in each case, the number corresponding to it as above.

The actual strains may be found as follows:—The actual strain on the end bay of the rafter next pier 1, is equal to half the weight of the loaded principal or the weight on one rafter, increased in the proportion of the length of the rafter to the rise, or as AC to CD in Fig. 27. If, therefore, the proportionate number for that bay (being $(N-1) \times a$, as given in Fig. 28), be called m , and the proportionate number for any other part be called n , the actual strain on the end bay multiplied by n and divided by m , gives the actual strain on that other part.

53. When the bottom tie of a roof principal is inclined, as is usually the case in practice, the general formulæ given in Fig. 28 are no longer applicable to the framing; and the strains may be then obtained as follows:—

In the roof shown in Fig. 29, the lower end of the

rafter at a is presumed to be supported by a wall or column in the usual manner; also the end b is sustained in its place by an equal pressure from the other rafter against it. It is therefore sufficient to deal with one-half of the roof; only observing that the tension on the central verticle bc must be doubled, as it receives an equal strain from each side.

As there are three bays in Fig. 29, each loaded point, d , e , and b , receives one-third the weight on the rafter.* To find the strains, set down df by scale to represent the weight at d ; draw fg parallel to the rafter, and gh to the tie; then the measure of dg by the same scale gives the strain on the strut brace; that of gf the compression on the rafter from the point a to d ; and that of gh the tension on the tie from the point a to k . The strut dk impresses on ke a tension measured by dh , which, together with one-third the weight on the rafter, is impressed on the point e . Set down el by scale equal to this pressure, draw lm parallel to the rafter and mn parallel to the tie; then the measure of lm gives a second strain on the rafter from a to e , that of nm a second strain on the tie from a to c , and that of em the strain on the strut brace ec . The strut ec then impresses on cb a tension (subsequently doubled as above explained), which is measured by en ; and which, together with the remaining third portion of the weight on the rafter, is impressed on the point b . Set down bo by scale equal to this pressure, and draw op parallel to the

* This supposition is not strictly correct for an uniform load, as the weight carried in that case at b , is less than that of a full bay. The difference, however, is immaterial in practice, and the effect is to increase slightly the calculated strains and strength of the structure.

tie, then the measures of $p o$ and $p b$ give the respective amounts of a third strain on the tie and rafter throughout. The strains upon each brace are thus obtained; and the additions of the strains above enumerated on the rafter and tie rod give the total strains on each of their bays.

54. If a roof be formed as shown in Fig. 30 and loaded at top, and if half the load be represented by $b c$, the comparative strains on each portion of the structure are as $a b$, $a c$, and twice $c d$ on the vertical $b c$; and when the bottom tie is horizontal, there is no strain on the vertical rod $b c$.

55. With the object of facilitating the calculation of these strains in the cases commonly occurring in practice, the diagrams in Figs. 31 to 34 are given in the plates. They represent the comparative strains on several roof framings, of which the rafters are inclined at a slope or pitch of $2\frac{1}{4}$ to 1, and the ties at a slope of 1 in 15, being the most approved proportions in practice. The span and weight of the entire principal in the diagrams are represented each by 1; and the diagram figures indicate the strains which result from thence on each part, and which are found in the manner above-mentioned in paragraph 53.

The actual strains on a principal corresponding in form with any of the above diagrams may be readily found thus:—Multiply the span of the roof by the width between two principals in feet, and the product by the weight per square foot of the loaded structure:—the result multiplied by the diagram numbers in succession gives the strain on each part in terms of the same weight.

56. The strains on the tie and rafter of a roof prin-

cipal are increased as the span and diminished as the depth at the centre; as in the case of an ordinary girder, of which it is a variety.

57. The author begs in conclusion to name the sources whence he has drawn early information or assistance on the subject of this treatise. His chief acknowledgments in this respect are due to the late Mr. Charles Heard Wild, the close of whose talented career by an early death has been to him the subject of unceasing regret. He has also to mention the benefit derived from the perusal of the well-known work of Mr. Edwin Clark on the Britannia Bridge; of an excellent treatise on Bracing by Mr. Robert Henry Bow; and of a paper on Diagonal Braces by Messrs. Doyne and Blood, read at the Institution of Civil Engineers.

Fig. 1. Paragraph 3.

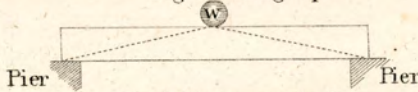


Fig. 2. Paragraph 5.

Load equally distributed

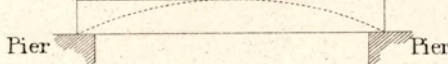


Fig. 3. Paragraphs 6.7.13.

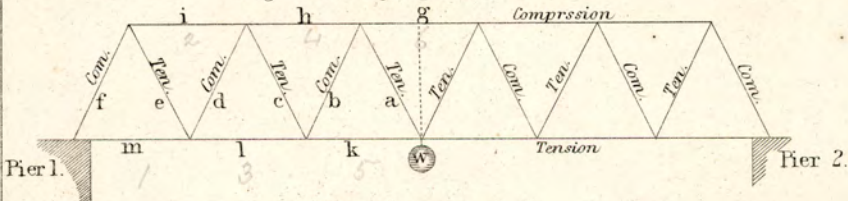


Fig. 4. Paragraph 8.

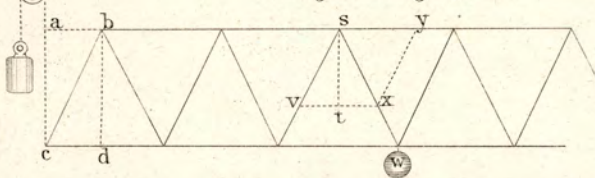


Fig. 5. Paragraph 15.

With Load on Top and End Braces Struts

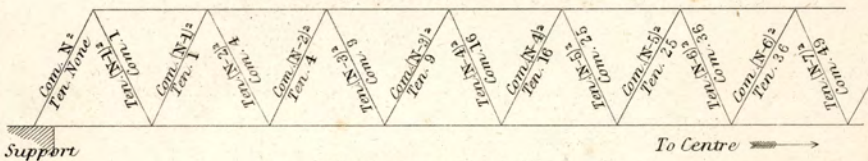


Fig. 6. Paragraph 15.

With Load on Top and End Braces Ties

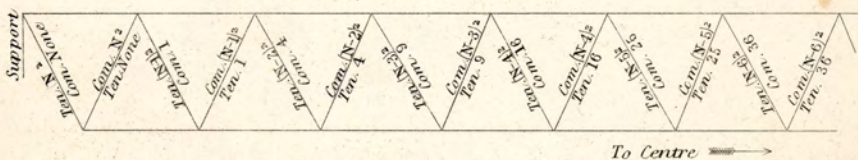


Fig. 7. Paragraph 15.

With Load at Bottom and End Braces Struts

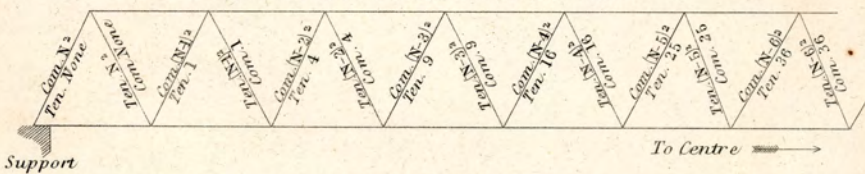


Fig. 8. Paragraph 15.

With Load at Bottom and End Braces Ties

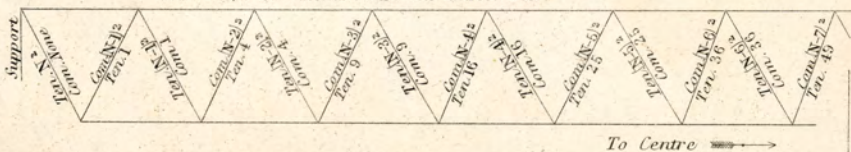


Fig. 9. Paragraphs 18.19.

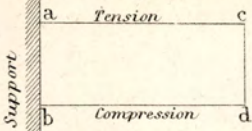


Fig. 10 Paragraph 18.

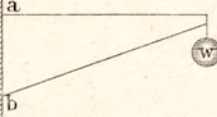


Fig. 11 Paragraph 19.

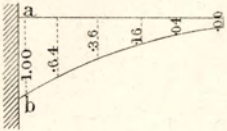


Fig. 12. Paragraphs 20.21.

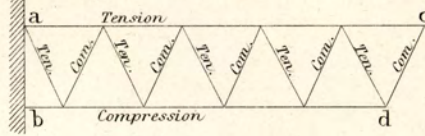


Fig. 13. Paragraphs 20.21.

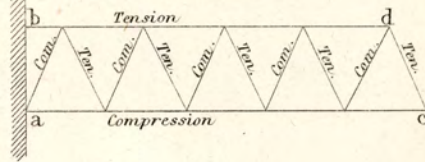


Fig. 14. Paragraph 22.

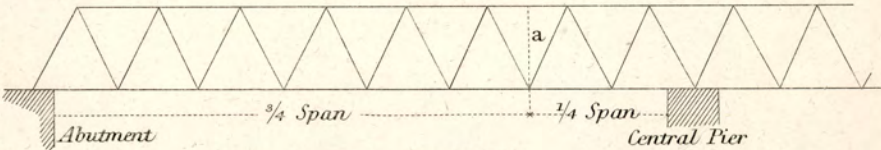


Fig. 15. Paragraph 22.

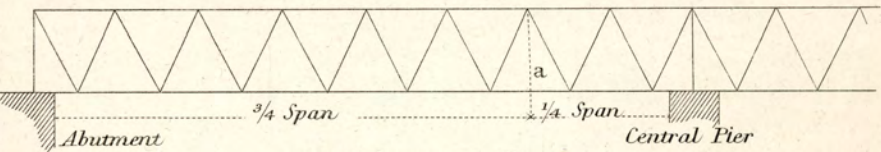


Fig. 16. Paragraph 22.

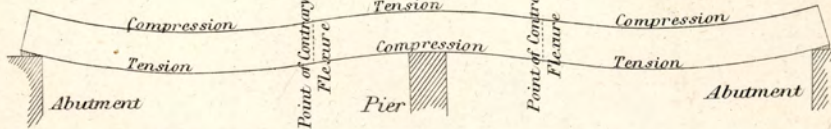


Fig. 17. Paragraph 26.

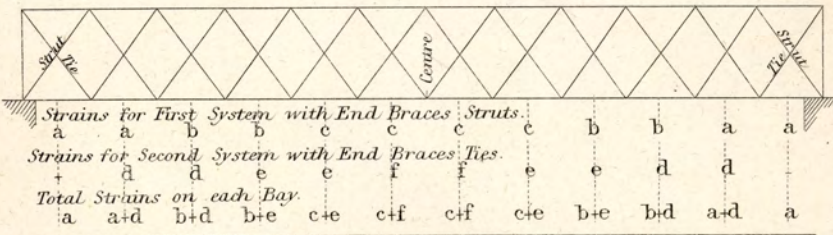
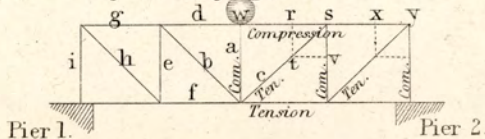


Fig. 18. Paragraph 28.



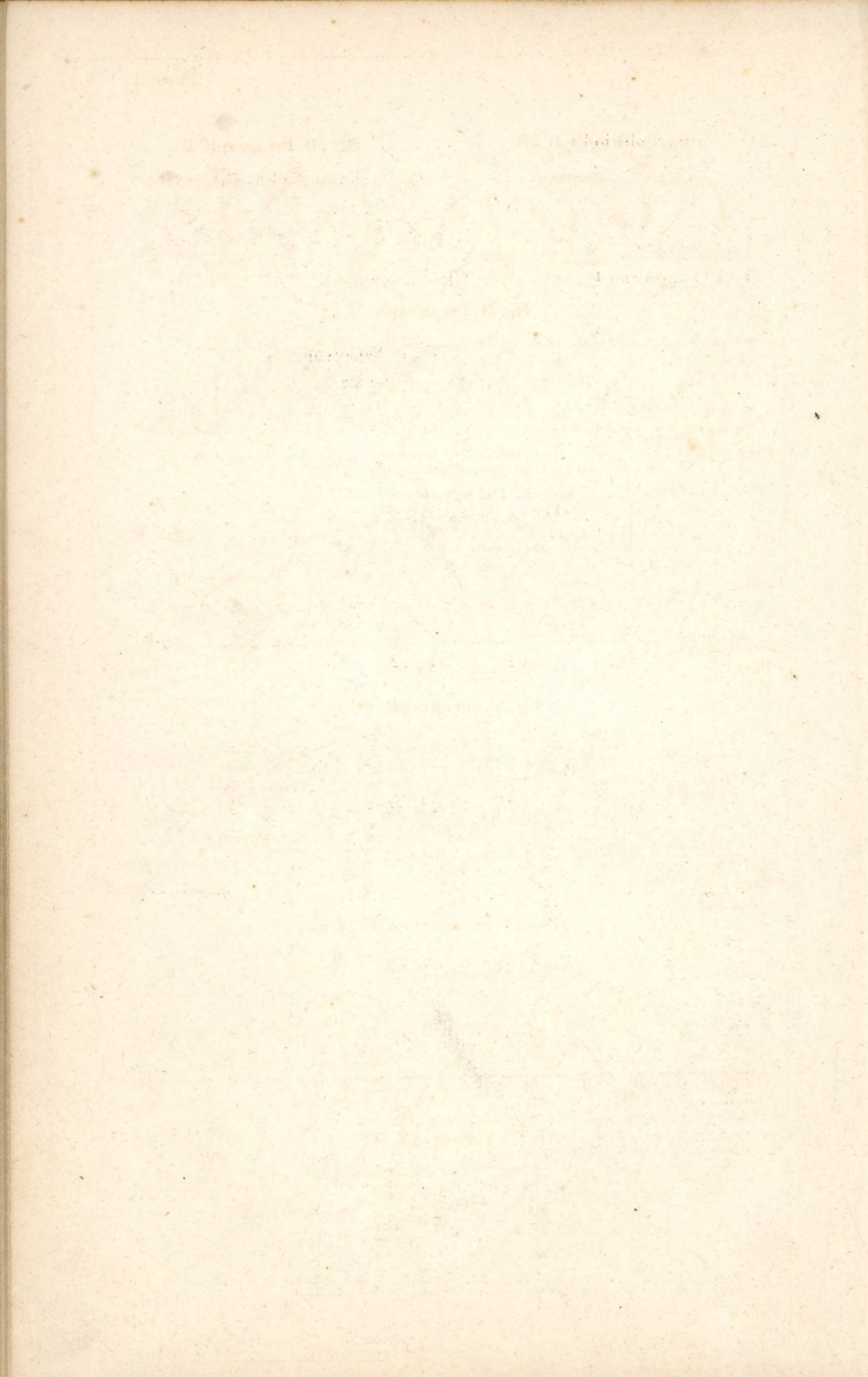


Fig. 19. Paragraph 30.

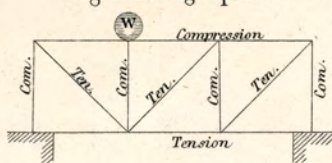


Fig. 20. Paragraph 31.

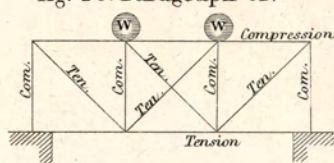


Fig. 21. Paragraph 33.

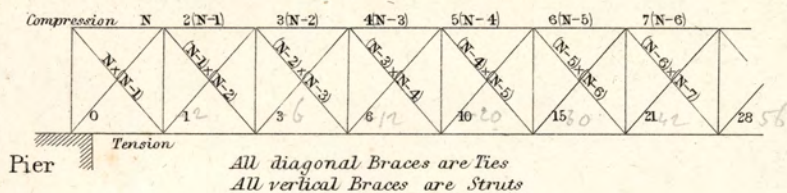


Fig. 22. Paragraphs 37. 38.

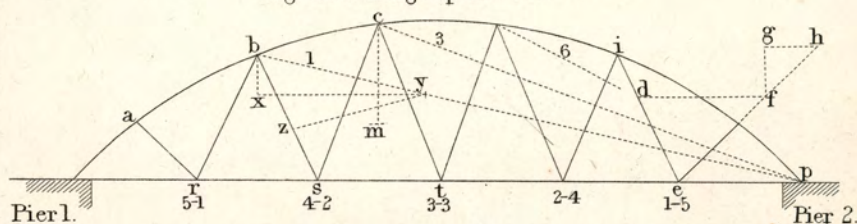


Fig. 23. Paragraph 39.

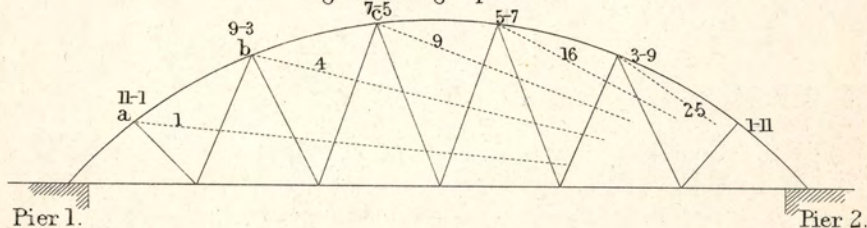


Fig. 24. Paragraphs 40. 41. 42.

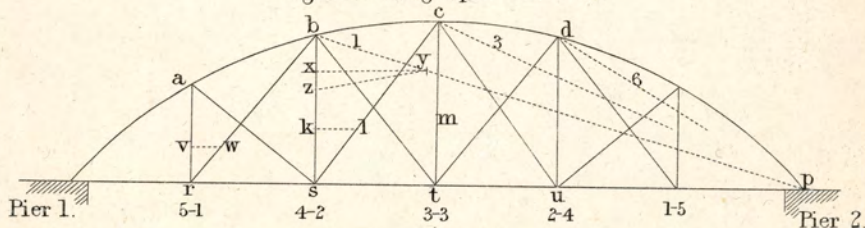
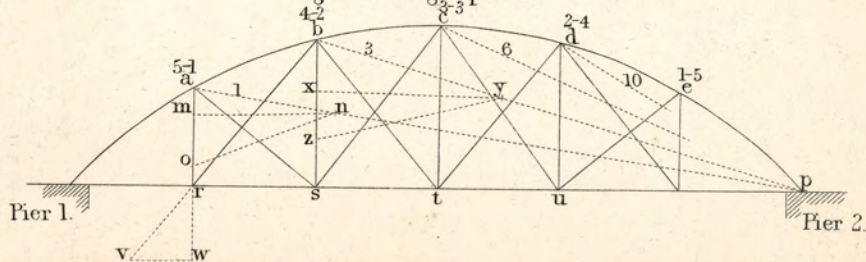


Fig. 25. Paragraph 43.



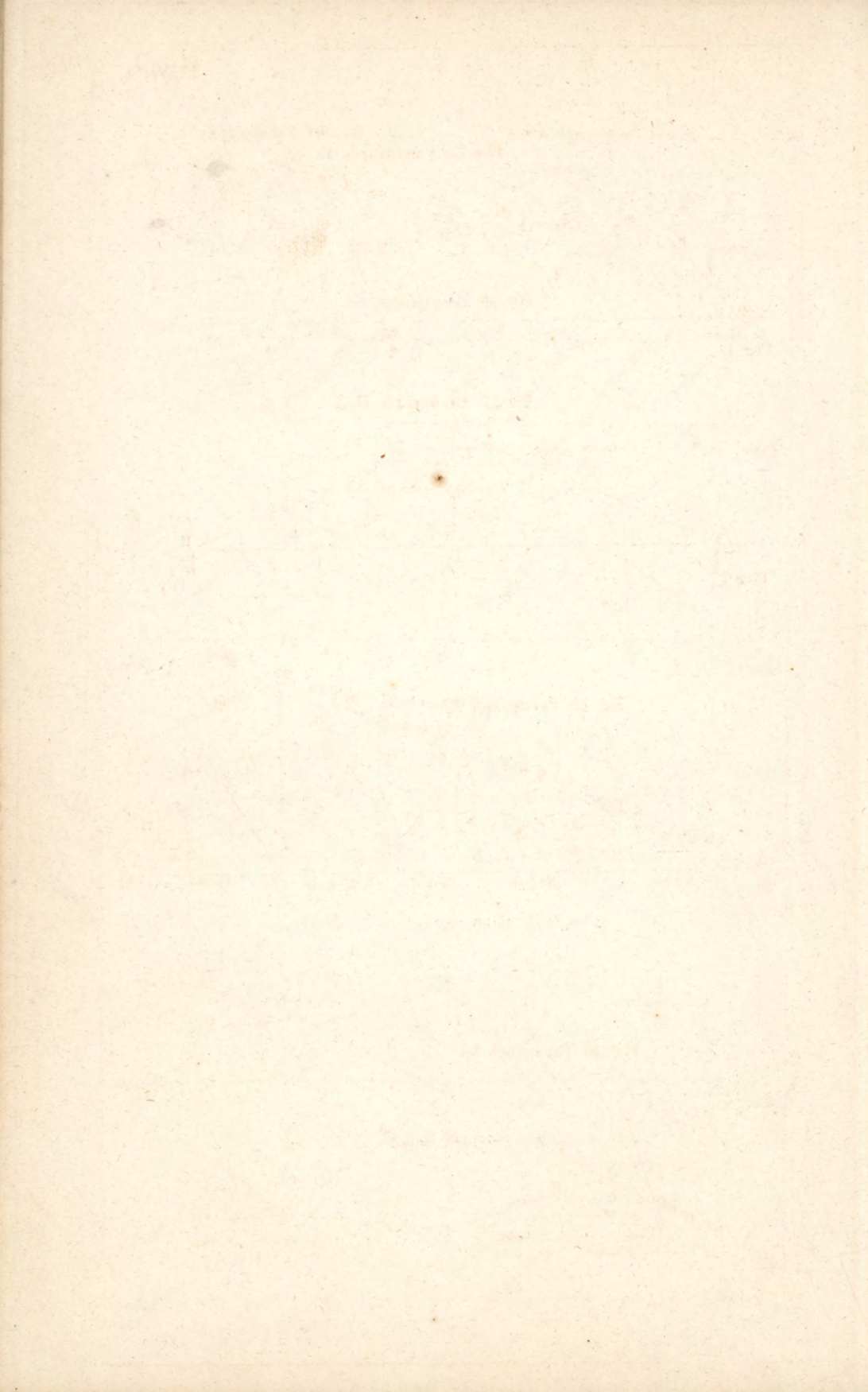


Fig. 26. Paragraph 45.

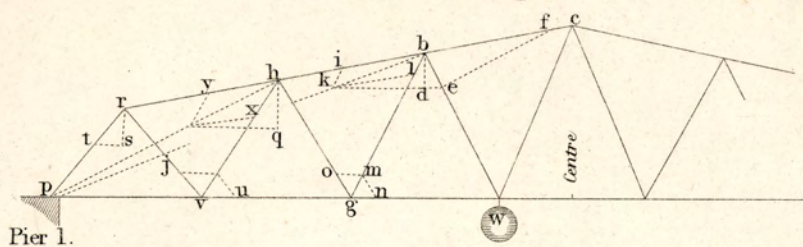


Fig. 27. Paragraph 51.

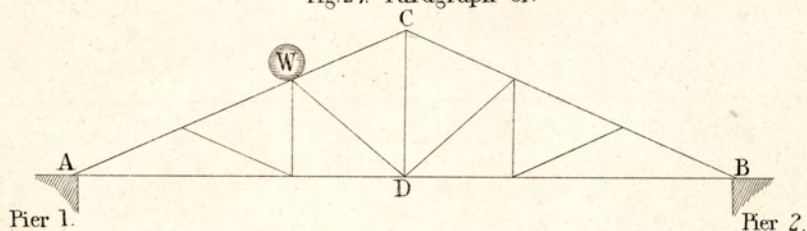


Fig. 28. Paragraph 52.

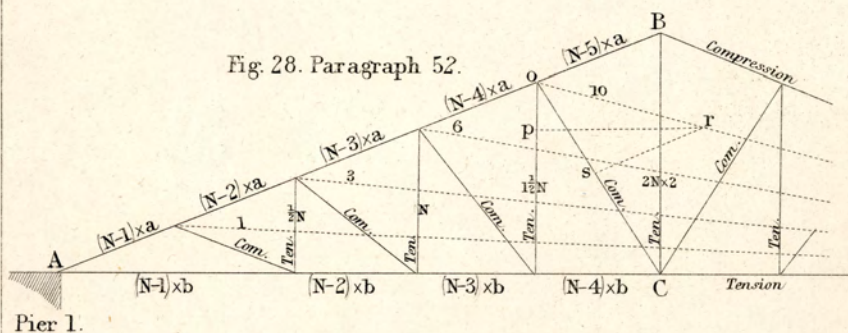


Fig. 29. Paragraph 53.

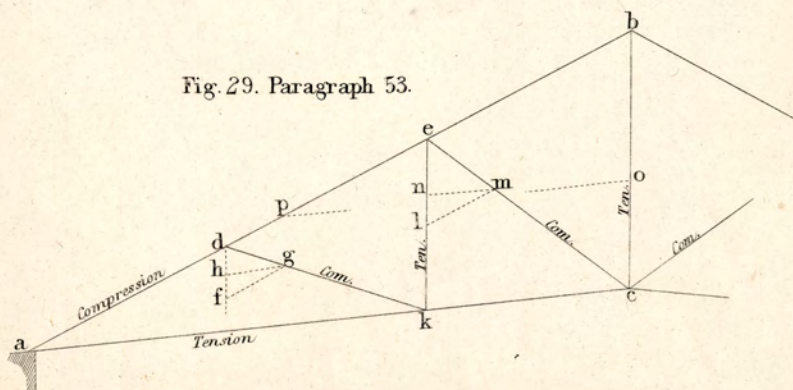


Fig. 30. Paragraph 54.

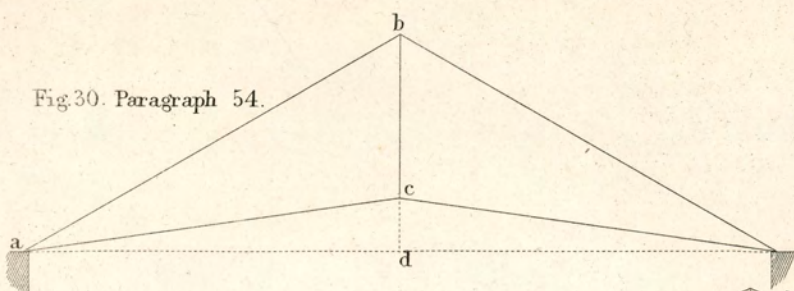


Fig. 31. Paragraph 55.

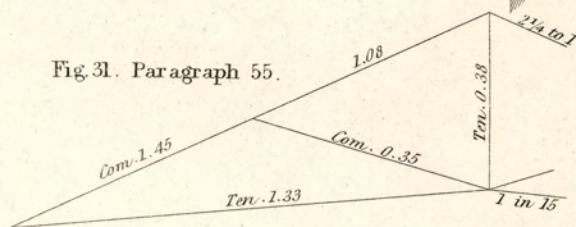


Fig. 32. Paragraph 55.

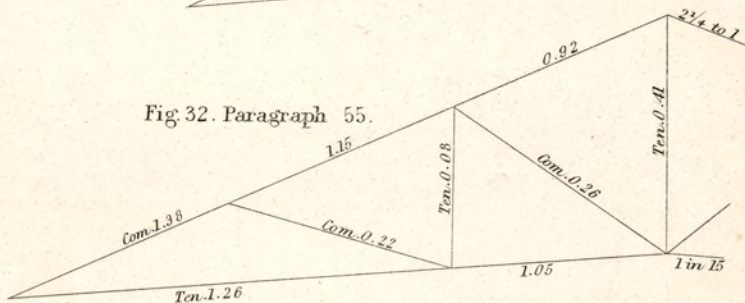


Fig. 33. Paragraph 55.

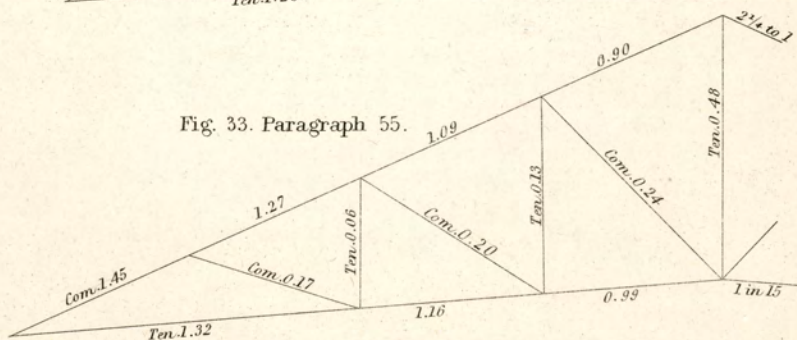
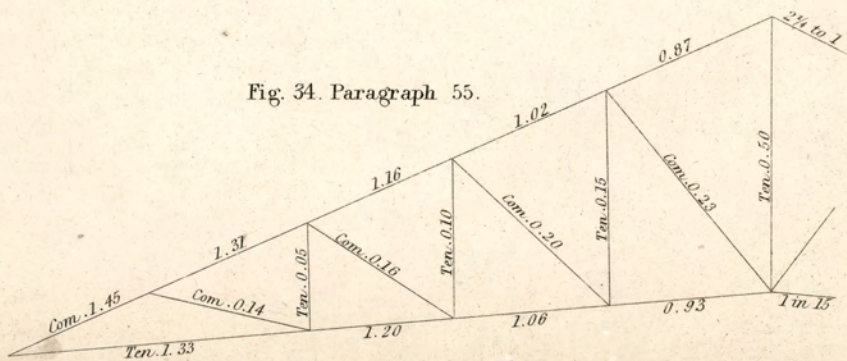
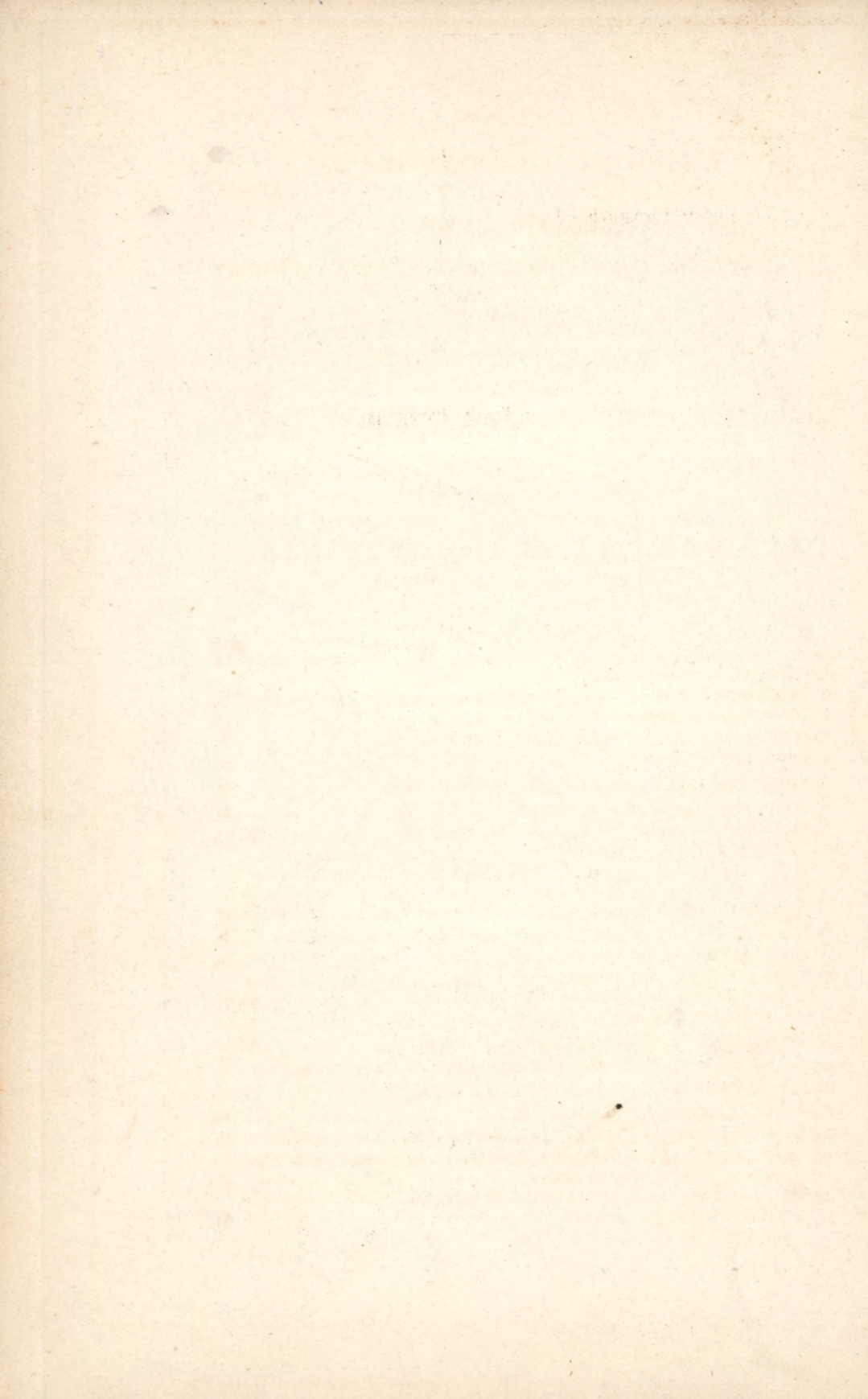


Fig. 34. Paragraph 55.





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